Philosophical Magazine Series 4
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/tphm15

I. On the relation between the radiating and absorbing powers of different bodies for light and heat

G. Kirchhoff

Version of record first published: 26 May 2009.

Frontispiece to *Phil. Mag.* *Ser.* 4. Vol. 20.

Richard Taylor

THE

LONDON, EDINBURGH AND DUBLIN

PHILOSOPHICAL MAGAZINE

AND

JOURNAL OF SCIENCE.

[FOURTH SERIES.]

JULY 1860.

I. *On the Relation between the Radiating and Absorbing Powers of different Bodies for Light and Heat.* *By* G. Kirchhoff*.

A BODY placed within a covering whose temperature is the same as its own, is unaffected by radiation, and must therefore absorb as many rays as it emits. Hence it has long been concluded that, at the same temperature, the ratio between the radiating and absorbing powers of all bodies is the same,—it being, however, assumed that bodies only emit rays of one kind. This law has been verified experimentally, especially by MM. de la Provostaye and Desains, in many cases in which the homogeneousness of the emitted rays could at least be so far assumed, inasmuch as they were all invisible. Whether the same law holds good when bodies emit rays of different kinds (which, strictly speaking, is always the case), has never hitherto been determined theoretically or by experiment. I have, however, now found that the law in question extends to this case also, provided that by the radiating power the intensity of one species of emitted rays be understood, and that the absorbing power be estimated with reference to rays of the same kind. Taken in this way, the ratio of the radiating and absorbing powers of all bodies at the same temperature is the same. I shall first give the theoretical proof of this principle, and then develope certain remarkable consequences that immediately follow therefrom, which partly explain phænomena already known, and partly suggest new ones.

All bodies emit rays, the quality and intensity of which depend on the nature and temperature of the bodies themselves. In addition to these, however, there may, under certain circumstances, be rays of other kinds,—as, for example, when a body

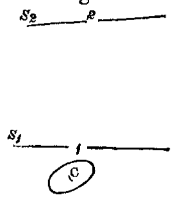
* Translated by Mr. F. Guthrie from Poggendorff's *Annalen*, vol. cix. p. 275.

is sufficiently charged with electricity, or when it is phosphorescent or fluorescent. Such cases are, however, here excluded.

When a body encounters rays from without, it absorbs a portion of them and converts it into heat. But, besides this species of absorption, there may, under certain circumstances, be others, as, for example, when a body is phosphorescent or fluorescent. It is, however, here assumed that all absorbed rays are converted into heat.

§ 1. Before the body C (fig. 1), imagine two screens, S_1, S_2, to be placed, containing openings 1 and 2, whose dimensions must be regarded as infinitely small in comparison with their distance apart, and each of which has a middle point. Through these two openings a pencil proceeds from the body C. Of this pencil let that part be considered which consists of waves, the length of which lies between λ and $\lambda+d\lambda$, and let this be divided into two component parts polarized in the perpendicular planes *a* and *b* passing through the axis of the pencil. Let the intensity of the part polarized in *a* be $Ed\lambda$: E is then the radiating power of the body.

Fig. 1.



Conversely, a pencil of rays polarized in plane *a*, and having waves of the length λ, falls on the body C through the openings 2 and 1. Of this, part is absorbed by the body, the rest being partly reflected and partly transmitted. Let the ratio of the absorbed to the incident rays be called A; then A will represent the power of absorption of the body.

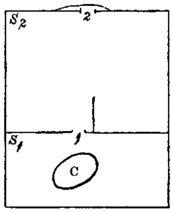
The magnitudes E and A depend on the nature and temperature of C, on the position and form of the openings 1 and 2, on the magnitude λ, and on the position of the plane *a*. It will be shown that the ratio of E to A is independent of the nature of the body; it will thence necessarily follow that it cannot be affected by the position of the plane *a*, and its independence of the position and form of the openings 1 and 2 will thence be easily deduced, so that it only remains to be determined how far it depends on the temperature of C and the wave-length λ.

The proof I am about to give of the law above stated, rests on the supposition that bodies can be imagined which, for infinitely small thicknesses, completely absorb all incident rays, and neither reflect nor transmit any. I shall call such bodies *perfectly black*, or, more briefly, *black* bodies. It is necessary in the first place to investigate the radiating power of bodies of this description.

§ 2. Let C be a black body. Let its radiating power (generally indicated by E) be called *e*. It will be shown that *e* remains the same when C is replaced by any other black body of the same temperature.

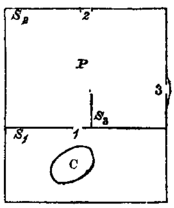
Imagine the body C enclosed in a black covering, of which the screen S_1 forms a part. Let the second screen, S_2, be black also, and let the two be united all round by black walls (fig. 2). Let the opening 2 be in the first place closed by a black surface, which I shall call surface 2, and let the whole system be kept at a constant temperature by being completely enclosed in a covering impermeable to heat, as, for example, in a perfectly reflecting surface. Then, since the temperature of the body C remains constant, the intensity of the incident rays (which, according to hypothesis, it entirely absorbs) must be equal to that of the emitted rays. Now imagine surface 2 to be removed and replaced by a portion of a perfectly reflecting spherical mirror having its centre in the middle point of opening 1. The equilibrium of the temperature of the system will be undisturbed, and the equality of the intensities of the emitted and incident rays must therefore still subsist. As, however, the body C emits the same rays as before, it follows that the intensity of the rays that impinge on it must be the same in both cases. By the removal of surface 2, those rays are withdrawn from the body which proceeded from that surface through opening 1. In the place of the rays so withdrawn, the mirror applied at 2 reflects back on the body the rays which proceeded from it through the openings 1 and 2*. Whence it may be concluded that the intensity of the pencil which proceeds from the body C through the openings 1 and 2, is equal to the intensity of the pencil which at the same temperature proceeds from surface 2 through opening 1. But this is independent of the form and constitution of the body C. The alleged law is therefore proved when all the rays of the pencils compared have the same wavelength λ, and are polarized in the same plane *a*. Regard, however, to the possible diversity of these rays renders somewhat more complex considerations necessary.

Fig. 2.



§ 3. In the arrangement represented in fig. 2, imagine a small plate P (fig. 3) placed between openings 1 and 2, which in the visible rays displays the colours of thin plates, and which, partly on account of its extreme thinness, and partly on account of its material constitution, neither absorbs nor emits any perceptible quantity of rays. Let this plate be so placed that it cuts the pencil passing between 1 and 2 at the

Fig. 3.



* The effect of the diffraction of the rays by the edges of opening 1 is here neglected. This is allowable if openings 1 and 2, though infinitely small in comparison with their distance apart, be considered as very great in comparison with the length of a wave.

angle of polarization, the plane of incidence being a. Let the wall which unites S_1 and S_2 be so situated that the image cast by plate P of opening 2 lies in it; and in the place and of the form of this image imagine an opening, which I shall call opening 3. Let opening 2 be closed by a black surface of the same temperature as the rest of the system, and let opening 3 be closed, in the first place by a similar surface (which I shall call surface 3), and secondly, by a perfect concave mirror, having its centre in the image which the plate P casts of opening 1. In both cases the equilibrium of temperature remains undisturbed, and for reasons similar to those mentioned in the last section; it thence follows that the sum of the intensity of the rays withdrawn from the body by the removal of surface 3, is equal to the sum of the intensities of the rays incident on the body in consequence of the application of the mirror. Let a black screen S_3, of the temperature of the rest of the system, be so placed that no rays emanating from opening 3 can reach the body directly. The first sum is then the intensity of the rays which proceed from surface 3, are reflected by plate P, and pass through the opening 1; let this be indicated by Q. The second sum consists of two parts: one depending on the body C, which is

$$=\int_0^\infty d\lambda e r^2,$$

where r indicates a magnitude depending on the constitution of plate P, and independent of λ; the second part consisting of rays which have proceeded from some portion of the black wall uniting S_1 and S_2, have penetrated plate P, and have been reflected, first by the mirror, and then by P. This portion I shall indicate by R. It is unnecessary further to determine the value of R, it being sufficient to observe that R, like Q, is independent of the nature of C. Between these magnitudes, then, there subsists the following equation:

$$\int_0^\infty d\lambda e r^2 + R = Q.$$

If, now, the body C be replaced by some other black body of the same temperature, e' indicating for this body what e does for the other, then

$$\int_0^\infty d\lambda e' r^2 + R = Q.$$

Whence

$$\int_0^\infty d\lambda (e - e') r^2 = 0.$$

Now let the index of refraction of plate P be supposed to be infinitely near unity. From the theory of the colours of thin plates, it follows that

$$r=\rho\sin^2\frac{p}{\lambda};$$

where p indicates a magnitude proportional to the thickness of the plate, and independent of λ, and ρ a magnitude independent of the thickness of the plate. The former equation then becomes

$$\int_0^\infty d\lambda(e-e')\rho^2\sin^4\frac{p}{\lambda}=0.$$

And since this equation holds good whatever be the thickness of plate P, that is, whatever be the value of p, it may be deduced that whatever λ may be,

$$e-e'=0.$$

In order to prove this, in the above equation for $\sin^4\frac{p}{\lambda}$ substitute its value

$$\tfrac{1}{8}\left(\cos 4\frac{p}{\lambda}-4\cos 2\frac{p}{\lambda}+3\right),$$

and differentiate twice with respect to p, we then have

$$\int_0^\infty d\lambda\,\frac{e-e'}{\lambda^2}\,\rho^2\left(\cos 4\frac{p}{\lambda}-\cos 2\frac{p}{\lambda}\right)=0.$$

Let $\frac{2}{\lambda}=\alpha$ and $(e-e')\rho^2=f\alpha$. Then

$$\int_0^\infty d\alpha\, f\alpha\,(\cos 2p\alpha-\cos p\alpha)=0.$$

And since when $\phi\alpha$ is any function of α,

$$\int_0^\infty d\alpha\,\phi\alpha\cos 2p\alpha=\tfrac{1}{2}\int_0^\infty d\alpha\,\phi\left(\frac{\alpha}{2}\right)\cos p\alpha,$$

as will be seen if $\frac{\alpha}{2}$ be substituted for α, the above equation may be written as follows:

$$\int_0^\infty d\alpha\left(f\frac{\alpha}{2}-2f\alpha\right)\cos p\alpha=0.$$

Multiply this equation by $dp\cos xp$, where x is any magnitude whatever, and integrate from $p=0$ to $p=\infty$. Then by Fourier's formula, that

$$\int_0^\infty dp\cos px\int_0^\infty d\alpha\phi(\alpha)\cos p\alpha=\frac{\pi}{2}\phi\lambda,$$

we get

$$f\left(\frac{x}{2}\right)=2fx,$$

or

$$f\left(\frac{\alpha}{2}\right)=2f\alpha.$$

From which it follows, either that $f\alpha$ is nothing for every value of α, or that it is infinitely great when α vanishes. But when α vanishes, λ becomes infinite. Recollecting then the meaning of $f\alpha$, and recollecting also that ρ is a proper fraction, and that neither e nor e' can become infinite when λ increases without limit, the second alternative cannot be admitted, and therefore for every value of λ we must have

$$e=e'.$$

§ 4. If the pencil which proceeds from the black body C through the openings 1 and 2 consisted partly of rays polarized in a plane, the plane of polarization of the polarized portion must rotate when the body itself rotates about the axis of the pencil. Such a rotation must therefore affect the value of e. But since, by the above equation, no such effect can be admitted, it follows that no part of the pencil can be so polarized. It can also be shown that no part of the pencil can be circularly polarized. We shall not give the proof of this here. Without this it will be admitted that black bodies can be imagined so constituted that there is no more reason why they should emit rays circularly polarized in one direction more than the other. All the black bodies hereafter mentioned are supposed to be of this kind, viz. that they emit no polarized rays whatever.

§ 5. The magnitude indicated by e depends, not only on the temperature and length of the wave, but also on the form and relative position of the openings 1 and 2. Let w_1 and w_2 be the projections of these openings on the planes which cut the pencil at right angles to its axis, and let s be the distance of the openings apart; then

$$e=\mathrm{I}\,\frac{w_1 w_2}{s_2},$$

where I is a function of the length of the wave and the temperature only.

§ 6. As the form of the body C is arbitrary, we may substitute for it a surface which exactly fills the opening 1, and which we shall call surface 1. The screen S_1 may then be imagined to be removed. The screen S_2 may also be removed, if the pencil to which e relates be defined as that which proceeds from surface 1 and is incident on surface 2, which exactly fills the opening 2.

§ 7. A consequence that immediately follows from the equation last obtained, and which will afterwards be made use of, is that openings 1 and 2 may be interchanged.

§ 8. We shall now establish a law which may be regarded as a generalization of that announced in the last section.

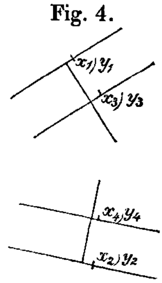
Between the two black surfaces 1 and 2 of equal temperature, imagine a body placed which may refract, reflect, and absorb the rays which pass between them in any way whatever. Several pencils may reach surface 2 from surface 1; of these let one be chosen, and let that part of it be taken when it leaves 1, which consists of waves of length between λ and $\lambda+d\lambda$, and let this be divided into two component parts polarized at right angles to each other in the planes a_1 and b_1. Let that part of the first component which reaches 2 be itself divided into two parts, whose planes of polarization are the perpendicular but otherwise arbitrary planes a_2, b_2. Let the intensity of the part polarized in a_2 be $\mathrm{K}d\lambda$. Of the pencil which pursues the same path, but in the opposite direction, viz. from 2 to 1, consider at 2 the part which consists of the waves whose length lies between λ and $\lambda+d\lambda$, and let it again be divided into two parts polarized in the planes a_2 and b_2, and let the portion of the first component part that reaches 1 be also divided into two components polarized in a_1 and b_1. Let the intensity of the part polarized in a_1 be $\mathrm{K}'d\lambda$. Then

$$\mathrm{K}=\mathrm{K}'.$$

The truth of this proposition shall, in the first place, be established on the hypothesis, first, that the rays suffer no diminution of intensity on their path, that is, that the refractions and reflexions to which they may be subjected cause no loss, and that there is no absorption; and secondly, that the rays that proceed from 1 polarized in a_1, impinge on 2 polarized in a_2, and conversely.

Through the middle point of 1 let a plane be placed perpendicular to the axis of the incident and emerging pencil, and in this plane let a system of rectangular coordinates be taken, of which the middle point of 1 is the origin. Let x_1, y_1 be the coordinates of some point in this plane (fig. 4). At the distance of unity from this plane let another plane be taken parallel to the first, and containing a system of coordinates parallel to the first system, having its origin in the axis of the pencil. Let x_3, y_3 be the coordinates of a point in this plane. Similarly, let a plane be taken passing through the middle point of 2, perpendicular to the axis of the incident and emerging pencil, and in this plane let a system of rectangular coordinates be taken having the middle point of 2 for its origin. Let x_2, y_2 be the coordinates of a point in this

Fig. 4.



plane. Lastly, at the distance of unity from the last plane, and parallel to it, let a fourth plane be taken, containing a system of rectangular coordinates parallel to those in the third plane, and having the axis of the pencil for its origin. Let x_4, y_4 be the coordinates of any point in this plane.

From the point $x_1 y_1$ a ray is supposed to proceed to the point $x_2 y_2$. Let the time it takes to pass from one point to the other be called T. Then T is some function of $x_1 y_1$, $x_2 y_2$, which we will suppose to be known. If the points $x_3 y_3$, $x_4 y_4$ lie in the path of the ray, and if for the sake of brevity the velocity of the ray *in vacuo* be taken as unity, then the time required to pass from $x_3 y_3$ to $x_4 y_4$ will be

$$=\mathrm{T}-\sqrt{1+\overline{x_1-x_3}^2+\overline{y_1-y_3}^2}$$
$$-\sqrt{1+\overline{x_2-x_4}^2+\overline{y_2-y_4}^2}.$$

If the points $x_3 y_3$, $x_4 y_4$ were given, and the points $x_1 y_1$, $x_2 y_2$ were required, they might be found from the condition that the above expression is a minimum. Supposing, therefore, that the eight coordinates, $x_1, y_1, x_2, y_2, x_3, y_3, x_4, y_4$, are very small, the condition that the four points of which they are the coordinates shall all lie in the path of the same ray is expressed by the following equations:—

$$x_3=x_1-\frac{\partial \mathrm{T}}{\partial x_1}, \quad x_4=x_2-\frac{\partial \mathrm{T}}{\partial x_2},$$
$$y_3=y_1-\frac{\partial \mathrm{T}}{\partial y_1}, \quad y_4=y_2-\frac{\partial \mathrm{T}}{\partial y_2}.$$

Now let $x_1 y_1$ be a point in the projection of surface 1 on the plane $x_1 y_1$, and let $dx_1\, dy_1$ be the element of this projection which contains the point $x_1 y_1$, and which must be considered as infinitely small as compared with the surfaces 1 and 2. Let $x_3 y_3$ be a point in a ray which proceeds from 1 to 2, $dx_3 dy_3$ the superficial element containing the point $x_3 y_3$, and of the same order of magnitude as $dx_1\, dy_1$. The intensity of the rays whose waves are of the length already mentioned, and which are polarized in the given plane, and proceed from $x_1 y_1$ through $x_3 y_3$, is then, according to § 5,

$$d\lambda \mathrm{I}\, dx_1\, dy_1\, dx_3\, dy_3.$$

Now according to the hypothesis we have assumed, the pencil arrives at 2 with its intensity undiminished, and forms an element of the magnitude indicated by K$d\lambda$. Whence for K we must have the integral

$$\mathrm{I}\int\int\int\int dx_1\, dy_1\, dx_3\, dy_3$$

taken between the proper limits.

The integration according to x_3 and y_3 must be between the values which those magnitudes have according to the equations above obtained, while x_1 and y_1 are constant, and x_2y_2 have all the different values which answer to the different points of the projection of surface 2 on the plane x_2y_2; the integrals according to $x_1\,y_1$ must then be taken over the projection of surface 1. The double integral,

$$\iint dx_3\,dy_3,$$

so limited is

$$=\iint\left(\frac{\partial x_3}{\partial x_2}\frac{\partial y_3}{\partial y_2}-\frac{\partial x_3}{\partial y_2}\frac{\partial y_3}{\partial x_2}\right)dx_2\,dy_2\,;$$

or applying the equations for x_3 and y_3,

$$=\iint\left(\frac{\partial^2 T}{\partial x_1\,\partial x_2}\cdot\frac{\partial^2 T}{\partial y_1\,\partial y_2}-\frac{\partial^2 T}{\partial x_1\,\partial y_2}\cdot\frac{\partial^2 T}{\partial y_2\,\partial x_1}\right)dx_2\,dy_2,$$

where the integrals are to be taken over the projection of surface 2. Whence

$$K=I\iiiint\left(\frac{\partial^2 T}{\partial x_1\,\partial x_2}\cdot\frac{\partial^2 T}{\partial y_1\,\partial y_2}-\frac{\partial^2 T}{\partial x_1\,\partial y_2}\cdot\frac{\partial^2 T}{\partial y_2\partial x_1}\right)dx_1\,dy_1\,dx_2\,dy_2,$$

where the integrals are to be taken over the projections of surfaces 1 and 2.

If the magnitude K′ be treated in the same way, it being remembered that a ray takes the same time to pass between two points down the same path in either direction, the same expression will be found for K′ as for K. The proposition to be proved is thus demonstrated, subject to the limitation already mentioned. This limitation may, however, be got rid of by means of an observation made by Helmholtz in his 'Physical Optics,' p. 169. Helmholtz here says (with somewhat different notation), "A ray of light proceeding from point 1 arrives at point 2 after suffering any number of refractions, reflexions, &c. At point 1 let any two perpendicular planes a_1, b_1 be taken in the direction of the ray; and let the vibrations of the ray be divided into two parts, one in each of these planes. Take similar planes a_2, b_2 in the ray at point 2; then the following proposition may be demonstrated. If when the quantity i of light polarized in the plane a_1 proceeds from 1 in the direction of the given ray, the part k thereof of light polarized in a_2 arrives at 2, then, conversely, if the quantity i of light polarized in a_2 proceeds from 2, the same quantity k of light polarized in a_2 will arrive at 1*."

* This proposition of Helmholtz ceases to hold good, as he himself observes, when the plane of polarization of the ray suffers any alteration such as that produced by magnetism, according to Faraday's discovery. In what follows, therefore, the effect of magnetic force must be excluded. Helmholtz limits his proposition also by the supposition that light suffers

Applying this proposition, and representing by γ the ratio $\frac{k}{i}$, in whichever direction the ray passes between the points $x_1 y_1$ and $x_2 y_2$, an expression is obtained for K and K′ which only differs from that above obtained by the occurrence of γ as a factor under the integral sign.

The equality of K and K′ therefore still subsists, even when γ has different values in the rays into which any one of the compared pencils may be considered as divided; it is, for example, unaffected if any part of the pencil be intercepted by a screen.

§ 9. The following proposition may also be proved of the same pencils as were compared in the last section. Of the pencil which proceeds from 1 to 2, consider at 2 that part which consists of waves whose length lies between λ and $\lambda d\lambda$, and let it be divided into two components polarized in a_2 and b_2. Let the intensity of the first of these components be $Hd\lambda$. Of the pencil that proceeds from 2 to 1, consider at 2 the part consisting of waves whose length lies between λ and $\lambda+d\lambda$, and divide this into two parts polarized in a_2 and b_2. Let the intensity of that portion of the first part which arrives at 1 be $H'd\lambda$. Then must

$$H=H'.$$

The proof of this proposition is as follows:—Let K and K′ have the same meaning as in the previous section, L and L′ being the magnitudes that K and K′ become when planes a_1 and b_1 are interchanged. Then $L=L'$, just as $K=K'$, and also

$$H=K+L;$$

for rays polarized perpendicularly to each other, provided they are parts of a non-polarized ray, do not interfere when they are brought back to a common plane of polarization; and, according to § 4, surface 1 emits none but non-polarized rays.

Lastly, we must have

$$H'=K'+L',$$

because two rays, whose planes of polarization are perpendicular, do not interfere. From these equations it follows that $H=H'$.

§ 10. Let fig. 2 have the same meaning as in § 3; only let the body C be no longer black, but a body of any kind. Let opening 2 be closed by surface 2; then a pencil proceeding from this surface through opening 1 reaches C, and is there partly absorbed and partly dispersed in various directions by reflexion and refraction. Of this pencil between 1 and 2 let that part be considered

no change of refrangibility such as occurs in fluorescence; this limitation, however, ceases to be necessary if, in the application of the proposition, only rays of a given length of wave are regarded.

which consists of waves whose length lies between λ and $\lambda+d\lambda$, and let it be divided into two components polarized in a and its perpendicular. Let that part of the first component which is not absorbed by C, and which therefore reaches the black covering in which the body C is enclosed, be $M'd\lambda$. Of the rays that proceed from the covering and are incident on C, a certain portion reach surface 2 through opening 1. The body C thus originates a pencil of rays which is incident on surface 2 through opening 1. Of this pencil consider that part which consists of waves whose length lies between λ and $\lambda+d\lambda$, and let this be divided into two parts polarized in plane a and its perpendicular. Let the intensity of the first component be $Md\lambda$. Then is

$$M=M'.$$

This follows from the proposition established in the last section, by applying that proposition to all the pencils which surface 2 and all the elements of the black covering exchange with each other through the medium of the body C, and then summing the equations so obtained.

§ 11. Let the arrangement represented in fig. 3, and described in § 3, be again taken, C, however, being no longer a black body, but one of any kind. In both the cases described in that section the equilibrium of temperature must still subsist, and therefore the *vis viva* that is withdrawn from the body by the removal of the surface 3, will be equal to the *vis viva* it receives by the application of the concave mirror. Let the letters used in § 3 have the same meaning as in that section, and let E and A have the signification given them in § 1. Then the *vis viva* withdrawn from the body C by the removal of surface 3 is, according to § 7,

$$=\int_0^\infty d\lambda e r A.$$

The *vis viva* which the body receives by means of the concave mirror consists of three parts:—The first due to the rays emitted by C itself; this is

$$=\int_0^\infty d\lambda E r^2 A.$$

The second due to the rays which, having proceeded from the part of the black covering opposite the mirror, have passed through the plate P, and have been reflected, first by the mirror, and then again by P. This, according to § 9, is

$$=\int_0^\infty d\lambda e r(1-r)A.$$

The third and last part is due to the rays which have fallen on C from various parts of the black covering, have been thence reflected or refracted through opening 1 towards surface 2, have been reflected by the plate P, and again by the mirror at 3, and, lastly, again reflected by P through opening 1. If M then have the meaning given it in § 10, this last part

$$=\int_0^\infty d\lambda \mathrm{M} r^2 \mathrm{A}.$$

It may appear doubtful whether the above expressions for the first and third of these portions are correct when C is in such a position that a finite portion of the pencil proceeding from 2 through opening 1, and incident on C, is by C reflected back towards 2. Such cases are therefore for the present excluded.

According to § 10, $\mathrm{M}=\mathrm{M}'$, and by definition $\mathrm{M}'=e(1-\mathrm{A})$. The third part is therefore

$$=\int_0^\infty d\lambda e(1-\mathrm{A}) r^2 \mathrm{A},$$

whence we have the equation

$$\int_0^\infty d\lambda (\mathrm{E}-\mathrm{A}e)\mathrm{A}r^2=0.$$

And from considerations identical with those mentioned in § 3 with reference to a similar equation, the conclusion may be drawn, that for every value of λ

$$\frac{\mathrm{E}}{\mathrm{A}}=e;$$

or, putting for e its value as obtained in § 5,

$$\frac{\mathrm{E}}{\mathrm{A}}=\mathrm{I}\frac{w_1 w_2}{s^2}.$$

The proposition we undertook to prove is therefore established, subject to the condition that no finite part of the pencil that proceeds from surface 2 through opening 1, and is incident on the body C, is reflected back by C to surface 2. That the proposition is true without this limitation, is obvious when we consider that, if the condition in question be not fulfilled, it is only necessary that the body C should be turned through an infinitely small angle in order to satisfy it, and that such a change of position can only cause an infinitely small change in the values of A and E.

The magnitude indicated by I is, as remarked in § 5, a function of the temperature and the wave-length. The determination of this function is a problem of the highest importance; and though difficulties stand in the way of our effecting this by experiment, there is nevertheless a well-grounded hope of ultimate success, since the form of the function in question is no doubt simple, as is the case with all functions hitherto discovered that do not depend on the properties of individual bodies. Whenever this problem is solved, the full fertility of the law above demonstrated will be apparent; even at present, however, important consequences may be deduced from it.

§ 12. If a body (a platinum wire, for example) be gradually heated up to a certain temperature, it only emits rays consisting of waves longer than those of the visible rays. Beyond that point, waves of the length of the extreme red begin to appear; and as the temperature rises, shorter and shorter waves are added; so that, for every temperature, rays of a corresponding length of wave are originated, while the intensity of the rays of greater wave-length is increased. If the law we have established be applied to this case, it will be seen that the function I, for waves of any given length, must vanish for all temperatures below that answering to the wave-length in question, and that, for temperatures above this, it must increase with the temperature.

Whence, applying the same proposition to other bodies, it follows that all bodies, when their temperature is gradually raised, begin to emit waves of the same length at the same temperature, and therefore become red-hot at the same temperature, emit yellow rays at the same temperature, &c.* The intensity of rays consisting of waves of a given length, which different bodies emit at the same temperature, may, however, be very different, since it is proportional to the power of absorption of the body for waves of that particular length. At the same temperature, accordingly, metal glows more brightly than glass, and glass more brightly than a gas. A body that remains perfectly transparent at the highest temperature never becomes red-hot. In a platinum ring of about 5 millims. diameter, I placed a small portion of phosphate of soda, and heated it in the dull flame of Bunsen's lamp. The salt melted, formed a fluid lens, and remained perfectly transparent; it, however, emitted no light, while the platinum ring, with which it was in contact, glowed brilliantly.

§ 13. For the same temperatnre the magnitude I is a continuous function of the wave-length, except for such values of the latter as render I evanescent. The truth of this assertion may be concluded from the continuity of the spectrum of a red-hot platinum-wire, provided it be admitted that the power of absorption of such a body is a continuous function of the length of the waves of the incident rays. It may also be affirmed, with the highest degree of probability, that while the temperature remains

* Draper, Phil. Mag. vol. xxx. p. 345; *Berl. Ber.* 1847.

constant the function I can have no strongly marked maxima and minima for waves of different lengths. Hence it follows that if the spectrum of a red-hot body presents discontinuities or strongly marked maxima or minima, the power of absorption of that body, regarded as a function of the length of the waves, must present similar discontinuities or strongly marked maxima and minima. Spectra with strongly marked maxima may be obtained by placing various salts in the flame of a Bunsen's lamp. Chloride of lithium affords interesting results in this respect. If a bead of this salt be melted in a platinum ring and placed in the mantle of the gas-flame, the spectrum of the flame (when unaffected by the presence of other salts and not too brilliant) is a single bright red line of light, formed of waves whose length is about the arithmetic mean of the lengths of the waves corresponding to the lines B and C of Fraunhofer. For waves of this length the radiating power of the flame is very considerable, while for waves of lengths corresponding to the other visible colours it is imperceptible. Accordingly, the power of absorption of the lithium-flame must be great for waves of this length, but very small for those constituting the other visible rays. If, therefore, a continuous spectrum be formed by suitable means and a lithium-flame be placed between the source of light and the slit of the apparatus, the spectrum is only affected in the place of the lithium line, its brightness being increased in that part by the radiation of the flame, while on the other hand it is diminished by its power of absorption for waves of that particular length. Suppose the absorptive power to be $\frac{1}{4}$. This would be the case, according to the law we have demonstrated, if the brightness of the line which constitutes the spectrum of the lithium-flame were $\frac{1}{4}$th of that of the corresponding line of the spectrum produced by a black body of the same temperature. The lithium-flame would then be without effect on the spectrum produced by any other source of light, provided the intensity of its own spectrum were $\frac{1}{4}$th of that of the corresponding line of the spectrum produced in its absence. If the source of light were proportionately brighter than this, the joint effect of it and the lithium-flame would be to produce a comparatively dull line on a bright ground; and conversely, if the source of light were proportionately duller, a bright line on a dull ground would become visible. In the first case the apparent dullness of the line would be greater in proportion to the brightness of the radiating body behind the lithium-flame; for in proportion as the light of the former was increased, so would that of the latter become less observable. For the particular value of the absorptive power mentioned, the brightness of the lithium line can, however, never be less than $\frac{3}{4}$ths of that of the surrounding parts of the spectrum.

But this limit may be decreased by increasing the thickness of the lithium-flame, and its consequent absorptive power.

A small bead of chloride of lithium, placed in the flame of a Bunsen's lamp, imparts to the latter so considerable an absorptive power for waves of the particular length mentioned, that if the rays of the sun be suffered to fall on the slit of the apparatus that forms the spectrum through such a flame, the corresponding part of the spectrum appears like a fine black line.

The spectra produced when other salts are placed in the flame are for the most part less simple than the lithium spectrum, and seldom exhibit such brilliant lines. *All* of them, however, are capable of being *reversed* by similar means. If flames of sufficient thickness be employed and light of suitable intensity be passed through them, the bright lines of the spectra may all be converted into lines of shade. The only exception would be in the case of a flame the light of which was partly produced by some immediate chemical action, or in case of a fluorescent flame. Experiment must decide whether such flames exist.

If the source of light employed is an incandescent body, the intensity of the light it emits depends on its temperature,—the intensity, for the same temperature, being greatest when the body is perfectly black. If this condition be fulfilled in the case of two sources of light, and if their temperature be the same, the spectrum of the one will be unaffected by the interposition of the other. The more remote source of light can therefore only reverse the spectrum of the other when it possesses a higher temperature, and the reversed spectrum will be more distinct the greater the excess of the temperature of the former source of light over that of the latter.

Besides the spectrum of the lithium-flame, I have succeeded in reversing that of the common salt-flame. This spectrum consists, as is well known, of two very brilliant yellow lines close together, the wave-length of which corresponds to Fraunhofer's double line D. If the rays of a Drummond light be passed through a salt-flame of not too high a temperature, the bright lines of the salt-spectrum become dark, and occupy the place of Fraunhofer's lines D, presenting in every respect the same appearance as those lines*.

§ 14. The wave-lengths which correspond to maxima of the radiating and absorbing powers are, as will be fully explained in another place, altogether independent of the temperature; and

* More recently Prof. Bunsen and I have likewise reversed the brighter lines of the spectra of potassium, calcium, strontium, and barium, by exploding before the slit of the spectral instrument mixtures of milk-sugar and chlorates of the respective metals during the passage of the sun's rays.—*May* 9, 1860.

moreover, in the case of salts which produce flames having such maxima, it is the *metal* that determines the nature of the spectrum. Imagine a body of very high temperature, in whose spectrum the double line D does not appear, surrounded by a gaseous atmosphere of somewhat lower temperature. If sodium be present in the latter, the spectrum of the whole system so constituted will contain the double line D. From the occurrence of these lines, the presence of sodium in the atmosphere may therefore be concluded. Now the sun is undoubtedly a body of this description*; and therefore, from the occurrence of the lines D in the solar spectrum, the presence of sodium in the sun's atmosphere may be concluded.

An objection may perhaps be urged against the justice of this conclusion. "The cause of the line D is," it may be said, "to be sought for in the atmosphere of the earth." This objection may, however, be disposed of on the following grounds:—

(1) The necessary quantity of sodium in the gaseous form can hardly be present in our atmosphere, and the gaseous form is necessary to produce the effect in question.

(2) If the line D depended on our atmosphere, it would become more strongly marked when the sun approached the horizon. I have, however, never observed any such change in the distinctness of these lines, though in the case of some of the neighbouring lines, such changes are very conspicuous.

(3) If the line D were not caused by the physical constitution of the sun itself, it would exist in the spectra of all the fixed stars of sufficient brightness; but according to Fraunhofer and Brewster, it is wanting in the spectra of some of the fixed stars though present in others.

The precise coincidence of the sodium lines with the D lines of Fraunhofer may be most satisfactorily proved by suffering the sun's rays to fall on the slit of the apparatus through a sodium-flame. The effect of the flame is exhibited in the increased distinctness, darkness, and breadth of the lines D. At the first glance it may appear somewhat strange that the sodium in a small flame should perceptibly increase the effect of the sodium present in the immense mass of the sun's atmosphere. Our surprise at this will, however, be diminished when we consider that the brightness of the lines D of the solar spectrum is determined by the temperature of the solar atmosphere, and especially of its outer portion, and that the temperature of this is certainly much greater than that of a gas-lamp. If a sodium-flame be imagined whose thickness may be regarded as infinite with respect to its power of absorbing the rays that correspond to the

* Whether the central mass of the sun, from which the light principally proceeds, is solid, liquid, or gaseous, may, as far as we are here concerned, be regarded as an open question.

lines D, and if rays from some other source of light be supposed to pass through this flame, and then to be separated into a spectrum, the brightness of the part of the spectrum corresponding to the lines D depends on the radiation of the flame alone. If, then, another sodium-flame of the same temperature be interposed, the spectrum remains unaltered; but if the flame interposed be of lower temperature, the lines must become duller. The effect on the solar spectrum of a gas-flame containing sodium is in this way accounted for, if it be admitted that the temperature of such a flame is lower than that of the outermost envelope of the sun's atmosphere; and this must certainly be the case, since the external portion of the sun's atmosphere cannot have a lower temperature than that of the focus of a powerful concave mirror directed towards the sun.

What has been stated concerning sodium is equally true of every other substance which, when placed in a flame of any sort, produces bright lines in its spectrum. If these lines coincide with the dark lines of the solar spectrum, the presence in the sun's atmosphere of the substances which produce them must be concluded, provided always that the lines in question cannot have their origin in the atmosphere of the earth. In this way means are afforded of determining the chemical constitution of the sun's atmosphere; and the same method even promises some information concerning the constitution of the brighter fixed stars*.

§ 15. From the proposition demonstrated in the first part of this essay, it follows that a body that absorbs more rays polarized in one plane than in another, must emit proportionately more rays of the first description than of the latter. Whence, as is known to be the case, a red-hot opake body with a smooth surface must emit rays in directions oblique to this surface partly polarized perpendicularly to the plane passing through the ray and normal to the surface; for of the incident rays polarized perpendicularly to the plane of incidence, the body reflects less and absorbs more than of the rays whose plane of polarization is the plane of incidence. By means of this principle the state of polarization of the emitted rays can easily be determined when the law of reflexion of the incident rays is known.

A tourmaline plate split perpendicularly to its optic axis absorbs at ordinary temperatures more of the perpendicularly

* In two communications laid before the Berlin Academy of Sciences on the 27th of October and the 15th of December, 1859, some statements are to be found concerning the physical constitution of the sun's atmosphere which are not introduced here. In the second of those communications also the proposition that forms the principal subject of this essay is proved in another way, but with less generality.

incident rays whose plane of polarization is parallel to the axis of the plate, than of those whose plane of polarization is perpendicular to that axis. If it be granted that the tourmaline retains this property when red-hot, then rays emitted in a direction perpendicular to its surface must be partly polarized in a plane passing through the optic axis, and therefore perpendicular to what is called the plane of polarization of the tourmaline. I have experimentally tested this striking deduction from the law here demonstrated, and have confirmed its truth. The tourmaline plates employed bore a considerable heat in a Bunsen's lamp for some time without suffering any permanent alteration, except that they appeared on cooling to have become a little cloudy at the edges. They retained the property of transmitting polarized light even when red-hot, though to a considerably less degree than at a lower temperature. This appeared on observing through a doubly-refracting prism, a red-hot platinum wire placed behind a tourmaline plate. The two images of the wire so produced were of unequal intensity, though the difference between them was much less than when observed through a plate of the ordinary temperature. To the doubly-refracting prism was then given that position in which the difference of the intensities of the two images was a maximum; if now it was the *upper* image of the wire that was the brightest, then on removing the wire and observing the plate alone, it was found that the *upper* image of the plate was unmistakeably though not strikingly *duller* than the other. The two images appeared exactly like two equal red-hot bodies, of which the upper possessed a lower temperature than the other.

§ 16. Place must be found for one more deduction from the law here established. If a space be entirely surrounded by bodies of the same temperature, so that no rays can penetrate through them, every pencil in the interior of the space must be so constituted, in regard to its quality and intensity, as if it had proceeded from a perfectly black body of the same temperature, and must therefore be independent of the form and nature of the bodies, being determined by the temperature alone The truth of this is obvious when we reflect that a pencil having the same position but the opposite direction to that chosen, is completely absorbed by the successive reflexions it undergoes from body to body. In the interior therefore of an opake red-hot body of any temperature, the illumination is always the same, whatever be the constitution of the body in other respects.

It may be observed, by the way, that the proposition demonstrated in this section does not cease to hold good even if some of the bodies are fluorescent. A fluorescent body may be defined as one whose radiating power depends on the rays incident on it for the time being. The equation $\frac{E}{A}=e$ cannot generally be true of such a body; but it is true if the body is enclosed in a black covering of the same temperature as itself, since the same considerations that led to the equation in question on the hypothesis that the body C was not fluorescent, avail in this case even if the body C be supposed to be fluorescent. To be convinced of this, it is only necessary to consider that if the magnitude E could have two different values in the two arrangements of the system represented in fig. 3, the difference of these values could only be an infinitely small quantity.

Heidelberg, January 1860.

*Postscript**.

1. Since the appearance of the above paper in Poggendorff's *Annalen*, I have received information of a prior communication closely related to mine. The communication in question is by Mr. Balfour Stewart, and appeared in the Transactions of the Royal Society of Edinburgh, vol. xxii. 1858. Mr. Stewart has made the interesting observation that a plate of rock-salt is much less diathermic for rays emitted by a mass of the same substance heated to 100° C., than for rays emitted by any other black body of the same temperature. From this circumstance he draws certain conclusions, and is led to a result similar to that which I have established concerning the connexion between the powers of absorption and emission. The principle enunciated by Mr. Stewart is, however, less distinctly expressed, less general, and not altogether so strictly proved as mine. It is as follows:—"The absorption of a plate equals its radiation, and that for every description of heat."

2. The fact that the bright lines of the spectra of sodium- and lithium-flames may be reversed, was first published by me in a communication to the Berlin Academy, October 27, 1859. This communication is noticed by M. Verdet in the February Number of the *Ann. de Chim. et de Phys.* of the following year, and is translated by Prof. Stokes in the March Number of the Philosophical Magazine. The latter gentleman calls attention to a similar observation made by M. Léon Foucault eleven years ago, and which was unknown to me, as it seems to have been to most physicists. This observation was to the effect that the electric arch between charcoal points behaves, with respect to the emission and absorption of rays of refrangibility answering to Fraunhofer's line D, precisely as the sodium-flame does according to my experiments. The communication made on this sub-

* Communicated by the Author.

ject by M. Foucault to the Soc. Philom. in 1849 is reproduced by M. Verdet, from the *Journal de l'Institut*, in the April Number of the *Ann. de Chim. et de Phys.*

M. Foucault's observation appears to be regarded as essentially the same as mine; and for this reason I take the liberty of drawing attention to the difference between the two. The observation of M. Foucault relates to the electric arch between charcoal points, a phænomenon attended by circumstances which are in many respects extremely enigmatical. My observation relates to ordinary flames into which vapours of certain chemical substances have been introduced. By the aid of my observation, the other may be accounted for on the ground of the presence of sodium in the charcoal, and indeed might even have been foreseen. M. Foucault's observation does not afford any explanation of mine, and could not have led to its anticipation. My observation leads necessarily to the law which I have announced with reference to the relation between the powers of absorption and emission; it explains the existence of Fraunhofer's lines, and leads the way to the chemical analysis of the atmosphere of the sun and the fixed stars. All this M. Foucault's observation did not and could not accomplish, since it related to a too complicated phænomenon, and since there was no means of determining how much of the result was due to electricity, and how much to the presence of sodium. If I had been earlier acquainted with this observation, I should not have neglected to introduce some notice of it into my communication, but I should nevertheless have considered myself justified in representing my observation as essentially new.

3. Since the above communication was printed in Poggendorff's *Annalen*, I have learned in the course of a written correspondence with Professor Thomson, that the idea was some years ago thrown out, if not published, that it might be possible, by comparing the spectra of various chemical flames with that of the sun and fixed stars, in the manner I have described, to become acquainted with the chemical constitution of the latter bodies (an idea now demonstrated to be correct by the observations and theoretical considerations above set forth). Prof. Thomson writes:—

"Professor Stokes mentioned to me at Cambridge some time ago, probably about ten years, that Professor Miller had made an experiment testing to a very high degree of accuracy the agreement of the double dark line D of the solar spectrum with the double bright line constituting the spectrum of the spirit-lamp burning with salt. I remarked that there must be some physical connexion between two agencies presenting so marked a characteristic in common. He assented, and said he believed a mechanical explanation of the cause was to be had on some such principles as the following:—Vapour of sodium must possess by its molecular structure a tendency to vibrate in the periods corresponding to the degrees of refrangibility of the double line D. Hence the presence of sodium in a source of light must tend to originate light of that quality. On the other hand, vapour of sodium in an atmosphere round a source, must have a great tendency to retain in itself, *i. e.* to absorb and to have its temperature raised by light from the source, of the precise quality in question. In the atmosphere around the sun, therefore, there must be present vapour of sodium, which, according to the mechanical explanation thus suggested, being particularly opake for light of that quality, prevents such of it as is emitted from the sun from penetrating to any considerable distance through the surrounding atmosphere. The test of this theory must be had in ascertaining whether or not vapour of sodium has the special absorbing power anticipated. I have the impression that some Frenchman did make this out by experiment, but I can find no reference on the point.

"I am not sure whether Professor Stokes's suggestion of a mechanical theory has ever appeared in print. I have given it in my lectures regularly for many years, always pointing out along with it that solar and stellar chemistry were to be studied by investigating terrestrial substances giving bright lines in the spectra of artificial flames corresponding to the dark lines of the solar and stellar spectra."

II. *Illustrations of the Dynamical Theory of Gases.* *By* J. C. Maxwell, *M.A., Professor of Natural Philosophy in Marischal College and University of Aberdeen.*

[Concluded from vol. xix. p. 32.]

Part II. *On the Process of Diffusion of two or more kinds of moving particles among one another.*

We have shown, in the first part of this paper, that the motions of a system of many small elastic particles are of two kinds: one, a general motion of translation of the whole system, which may be called the motion in mass; and the other a motion of agitation, or molecular motion, in virtue of which velocities in all directions are distributed among the particles according to a certain law. In the cases we are considering, the collisions are so frequent that the law of distribution of the molecular velocities, if disturbed in any way, will be re-established in an inappreciably short time; so that the motion will always con-